An object that moves in a circle at constant speed, \( v \), is said to experience uniform circular motion (UCM). The magnitude of the velocity remains constant, but the direction of the velocity is continuously changing, as shown in Figure 1. Notice that the velocity vectors are not pointing in the same direction. The velocity vector is always tangent to the circular path. This is why, should the string break, the object flies off in a straight line.

An object revolving in a circle is continuously accelerating, even when the speed remains constant. The acceleration vector is always pointing toward the center of the circular path and it is always perpendicular to the velocity vector, as shown in Figure 2. **Centripetal** (center-seeking) **acceleration**, \( a_c \), is defined as:

\[
a_c = \frac{v^2}{r}
\]

If we measure the period, \( T \), the time needed for the object to make a complete revolution, we can calculate the speed of the object in UCM. During this time, it travels a distance equal to the circumference of the circle, \( 2\pi r \), where \( r \) is the radius of the circular path. The object’s speed, then, is represented by \( v = \frac{2\pi r}{T} \).

According to Newton’s second law (\( \Sigma F = ma \)), an object that is accelerating must have a net force acting upon it and we call this force **centripetal force**. This force is acting in the same direction as the acceleration which is toward the center of the circle (Figure 3). The equation for centripetal force can be expressed as

\[
F_c = ma = ma_c = m \left[ \frac{v^2}{r} \right] = m \left[ \frac{\left( \frac{2\pi r}{T} \right)^2}{r} \right] = m \left( \frac{4\pi^2 r}{T^2} \right)
\]

Centripetal force is the net force which points toward the center of the circle, and, as examples, can take the form of a gravitational force, frictional force, or tension in a string.
PURPOSE
In this activity you will verify the relationship between centripetal force, mass, and velocity in an object in uniform circular motion. You will be using the assumption that the centripetal force generated by the whirling stopper balances the force of weight for the hanging mass.

MATERIALS
- centripetal force apparatus (Figure 4)
- nylon cord or thick monofilament fishing line
- 2-hole stoppers of different sizes
- meter stick
- several large washers or hooked masses
- stopwatch
- large paper clip
- string for tying the stacks of washers together

SAFETY ALERT
1. Goggles must be worn at all times.
2. Make sure you have enough room to swing your stopper without hitting other students.
3. Check to see that your cord is not frayed before beginning your experiment.

PROCEDURE
1. Put your goggles on and leave them on for the entire laboratory period.
2. One partner should construct your apparatus as pictured in Figure 4 below.
a. Mass the rubber stopper and record its mass in kilograms in Data Table 1 on your student answer page. Attach one end of the cord securely to the rubber stopper.

b. Pass the other end of the cord through the glass or plastic tube.

c. If using hooked masses, tie them to the string as shown. Otherwise, bend a large paperclip into a “hook” and attach it to the free end of the cord. This hook needs to support several large washers.

3. The other partner should use a balance to mass your group’s washers + hook. Keep adding washers until the mass exceeds 100 grams. Record the mass of your stack of washers and hook in kilograms in Data Table 1 on your student answer page.

4. Load the stack of washers onto the hook. If the stack is too large to hang securely from the hook, you can secure the stack together with a piece of string and hang the string from the hook. If you choose this method, re-weigh your hook as well as the stack of washers with the string attached.

5. Adjust the cord so that there is about 0.75 m of cord between the top of the tube and the stopper.

6. The partner that is going to operate the apparatus should support the stack of washers or hanging mass in one hand and hold the tube with the other. Begin whirling the stopper by moving the tube in a circular motion. Slowly release the stack of washers and adjust the speed of the stopper so that the stack of washers remains relatively stationary, meaning the stack is no longer climbing or falling. Important: Keep the stopper whirling in an elevated, horizontal circle as pictured in Figure 4. The tube should be held high enough in the air so that the spinning stopper does not threaten any part of the spinner’s head. Make several trial runs to master the technique. Be mindful of your space so that you do not accidentally hit any other students with your spinning stopper.

7. When you have learned how to keep the velocity of the stopper and the position of the washers relatively constant, have your partner measure the time it takes for 20 revolutions by counting down the revolutions “5, 4, 3, 2, 1, go! 1, 2, 3, ….” Record this time in Data Table 1 on your student answer page.

8. The partner with the apparatus should stop the whirling stopper by placing his or her finger at the top of the tube so as to capture the length of the radius of your circle. Use caution when performing this task so as not to be hit by the stopper. Once the stopper comes to rest, keep your finger in place so that the length of the cord will not change.

9. Measure the radius, \( r \), from the center of the tube to the center of the stopper. Use the appropriate number of significant digits and record the value in Data Table 1 on your student answer page.

10. Repeat the procedure for two additional trials keeping the same stopper and the same stack of washers, but varying the radius. Keep the radius between 0.50 m and 0.90 m. Record all data to the appropriate number of significant digits in your data table.
11. Repeat the procedure for three additional trials keeping the same radius [you may need to mark your cord at the point where it exits the tube so you can have a visual cue that your radius is consistent] but varying the size of your stopper. Record all data to the appropriate number of significant digits in your data table.

12. Repeat the procedure for a final three trials keeping the same radius and same stopper, but varying the mass of your washer stack. Record all data to the appropriate number of significant digits in your data table.
# Centripetal Force

## Exploring Uniform Circular Motion

### DATA AND OBSERVATIONS

<table>
<thead>
<tr>
<th>Trial</th>
<th>Mass of Washers (kg)</th>
<th>Mass of Stopper (kg)</th>
<th>Total Time (s)</th>
<th>Radius (m)</th>
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### ANALYSIS

1. Calculate the force of weight, $F_w$, of the hanging mass and enter it in Table 2 as centripetal force, $F_c$.

$$ F_w = mg = F_c \quad \text{and} \quad g = 9.8 \frac{m}{s^2} $$

2. Calculate the period of revolution by dividing the total time by the number of revolutions and enter it in Table 2.

$$ T = \frac{\text{time (s)}}{20 \text{ revolutions}} $$
3. Calculate the circumference of revolution from the radius and enter it in Table 2.

\[ \text{circumference} = \pi d = 2\pi r \]

4. Use the circumference and period to find the velocity and enter it in Table 2.

\[ v = \frac{\text{circumference}}{T} \]

<table>
<thead>
<tr>
<th>Trial</th>
<th>Centripetal Force, (F_c) (N)</th>
<th>Period, (T) (s)</th>
<th>Circumference (m)</th>
<th>Velocity (m/s)</th>
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**CONCLUSION QUESTIONS**

1. Survey the data in Table 2. What is the relationship between the velocity of a whirling object and the centripetal force that is exerted on it?

2. What is the relationship between the radius of revolution and the velocity of an object in uniform circular motion?
3. What is the relationship between the mass and velocity of an object in uniform circular motion?

4. Step one of the analysis section assumes that $F_W = F_c$ by using the assumption that the centripetal force generated by the whirling stopper balances the force of weight for the hanging mass. What additional assumption is made regarding the value for the centripetal acceleration?

5. A student neglects to reweigh the stack of washers after tying them together before performing one of the trials. How will this error affect the reported centripetal force? State clearly whether the centripetal force increases, decreases, or remains the same and mathematically justify your answer.

6. A 13.0-g rubber stopper is attached to a 0.93-m string. The stopper is swung in a horizontal circle, making one revolution in 1.18 s. Calculate the centripetal force exerted by the string on the stopper.

7. A coin is placed on a vinyl stereo record making 33½ revolutions per minute.
   a. In what direction is the acceleration of the coin?
   
   b. Calculate the magnitude of the acceleration when the coin is placed 10.0 cm from the center of the record.
8. A carnival ride has a 2.0 m radius and rotates once each 0.90 s.
   a. Calculate the speed of a rider at this radius.
   b. Calculate the centripetal acceleration of the rider.

9. As you whirled the mass in a circular path, you did your best to keep it swinging in a horizontal circle. Is it possible to keep the mass whirling in a perfectly horizontal circle (parallel to the floor)? Explain your answer and draw and label the forces acting on the mass from a side view using the diagram below.

10. A student tries to swing a stopper of mass 40 grams in a horizontal circle, but finds that the stopper “droops” down a little due to the gravitational force acting on the stopper, as shown in the figure below. The string has a length \( L = 0.50 \text{ m} \), and is swung so that it remains an angle of \( \theta = 60^\circ \) from the vertical.
a. On the diagram above, draw and label all of the forces acting on the stopper as it travels around the circle. (Remember: centripetal force is not an “extra” force that acts on the mass, but is the sum of the forces which point toward the center of the horizontal circle.)

b. Can we say that the tension in the string is the centripetal force? Explain your answer.

c. Show the horizontal and vertical components of the tension force on the diagram above.

d. Determine the vertical component $T_y$ of the tension in the string.

e. Determine the horizontal component $T_x$ of the tension in the string.

f. Determine the tangential speed $v$ necessary to keep the mass revolving in this circle.